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On the Motion of a Vortex Filament in an External Flow

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Abstract

We consider a nonlinear model equation describing the motion of a vortex filament immersed in an incompressible and inviscid fluid. In the present problem setting, we also take into account the effect of external flow. We prove the unique solvability, locally in time, of an initial value problem posed on the one dimensional torus. The problem describes the motion of a closed vortex filament.

1 Introduction

A vortex filament is a space curve on which the vorticity of the fluid is concentrated. Vortex filaments are used to model very thin vortex structures such as vortices that trail off airplane wings or propellers. In this paper, we prove the solvability of the following initial value problem which describes the motion of a closed vortex filament.

$$
\begin{cases} \boldsymbol{x}_t = \dfrac{\boldsymbol{x}_\xi \times \boldsymbol{x}_{\xi\xi}}{|\boldsymbol{x}_\xi|^3} + \boldsymbol{F}(\boldsymbol{x}, t), & \xi \in \mathbf{T},\ t > 0, \\ \boldsymbol{x}(\xi, 0) = \boldsymbol{x}_0(\xi), & \xi \in \mathbf{T}, \end{cases} \tag{1.1}
$$

where $\boldsymbol{x}(\xi,t) = (x_1(\xi,t), x_2(\xi,t), x_3(\xi,t))$ is the position vector of the vortex filament parametrized by ξ at time t, the symbol $\times$ is the exterior product in the three dimensional Euclidean space, $\boldsymbol{F}(\cdot,t)$ is a given external flow field, $\mathbf{T}$ is the one dimensional torus $\mathbf{R}/\mathbf{Z}$, and subscripts are differentiations with the respective variables. Problem (1.1) describes the motion of a closed vortex filament under the influence of external flow. Such a setting can be seen as an idealization of the motion of a bubblering in water, where the thickness of the ring is taken to be zero and some environmental flow is also present. Many other phenomena can be modeled by a vortex ring or a closed vortex filament and are important in both application and theory. Here, we make the distinction between a vortex ring and a closed vortex filament. A vortex ring is a closed vortex tube, in the shape of a torus, which has a finite core thickness. A closed vortex filament is a closed curve, which can be regarded as a vortex ring with zero core thickness.

The equation in problem (1.1) is a generalization of a equation called the Localized Induction Equation (LIE) given by

$$\boldsymbol{x}_t = \boldsymbol{x}_s \times \boldsymbol{x}_{ss},$$

which is derived by applying the so-called localized induction approximation to the Biot–Savart integral. Here, s is the arc length parameter of the filament. The LIE was first derived by Da Rios in 1906 and was re-derived twice independently by Murakami et al. in 1937 and by Arms and Hama in 1965. Many research has been done on the LIE and many results have been obtained. Nishiyama and Tani [11, 12] proved the unique solvability of the initial value problem in Sobolev spaces. Koiso considered a geometrically generalized setting in which he rigorously proved the equivalence of the LIE and a nonlinear Schrödinger equation. This equivalence was first shown by Hasimoto [6] in which he studied the formation of solitons on a vortex filament. He defined a transformation of variable known as the Hasimoto transformation to transform the LIE into a nonlinear Schrödinger equation. The Hasimoto transformation was proposed by Hasimoto [6] and is a change of variable given by

$$\psi = \kappa \exp\left(\mathrm{i}\int_0^s \tau \,\mathrm{d}s\right),$$

where κ is the curvature and τ is the torsion of the filament. Defined as such, it is well known that ψ satisfies the nonlinear Schrödinger equation given by

$$\mathrm{i}\frac{\partial\psi}{\partial t} = \frac{\partial^2\psi}{\partial s^2} + \frac{1}{2}|\psi|^2\psi. \tag{1.2}$$

The original transformation proposed by Hasimoto uses the torsion of the filament in its definition, which means that the transformation is undefined at points where the curvature of the filament is zero. Koiso [9] constructed a transformation, sometimes referred to as the generalized Hasimoto transformation, and gave a mathematically rigorous proof of the equivalence of the LIE and (1.2). More recently, Banica and Vega [2, 3] and Gutiérrez, Rivas, and Vega [4] constructed and analyzed a family of self-similar solutions of the LIE which forms a corner in finite time. The authors [1] proved the unique solvability of an initial-boundary value problem for the LIE in which the filament moved in the three-dimensional half space. Nishiyama and Tani [11] also considered initial-boundary value problems with different boundary conditions. These results fully utilize the property that a vortex filament moving according to the LIE doesn't stretch and preserves its arc length parameter. This is not the case when we consider external flow.

The LIE can be naturally generalized to take into account the effect of external flow. The model equation is given by

$$\boldsymbol{x}_t = \frac{\boldsymbol{x}_\xi \times \boldsymbol{x}_{\xi\xi}}{|\boldsymbol{x}_\xi|^3} + \boldsymbol{F}(\boldsymbol{x}, t). \tag{1.3}$$

Here, the parametrization of the filament has been changed to ξ because, unlike the LIE, a vortex filament moving according to (1.3) stretches in general and the arc length is no longer preserved. It is worth mentioning that if the Jacobi matrix of $\boldsymbol{F}$ is skew-symmetric, which amounts to assuming that the effect of external flow consists only of translation and rigid body rotation, then the solvability for (1.3) can be considered in the same way as for the LIE. This is because if the Jacobi matrix is skew-symmetric, then the filament no longer can stretch, and the techniques used in the analysis of the LIE can be utilized for (1.3). Thus, in what follows, we don't assume any structural conditions on $\boldsymbol{F}$.

Regarding the solvability of (1.3), Nishiyama [10] proved the existence of weak solutions to initial and initial-boundary value problems in Sobolev spaces. The solutions obtained by Nishiyama are weak in the sense that the uniqueness of the solution is not known, but the equation is satisfied in the point wise sense almost everywhere. The result presented in this paper is an extension of Nishiyama's result for the initial value problem, and we proved the unique solvability in higher order Sobolev spaces.

The contents of the rest of the paper are as follows. In Section 2, we define notations used in this paper and state our main theorem. In Section 3, we give a brief description for the construction of the solution, and in Section 4, we give the main part of the proof of the theorem, which is to obtain energy estimates of the solution in $C([0,T];H^m(\mathbf{T}))$, in more detail.

2 Function Spaces, Notations, and Main Theorem

We define some function spaces that will be used throughout this paper, and notations associated with the spaces. For a non-negative integer m, and $1 \leq p \leq \infty$, $W^{m,p}(\mathbf{T})$ is the Sobolev space containing all real-valued functions that have derivatives in the sense of distribution up to order m belonging to $L^p(\mathbf{T})$. We set $H^m(\mathbf{T}) := W^{m,2}(\mathbf{T})$ as the Sobolev space equipped with the usual inner product. The norm in $H^m(\mathbf{T})$ is denoted by $\|\cdot\|_m$ and we simply write $\|\cdot\|$ for $\|\cdot\|_0$. Otherwise, for a Banach space X, the norm in X is written as $\|\cdot\|_X$. The inner product in $L^2(\mathbf{T})$ is denoted by $(\cdot,\cdot)$.

For $0 < T < \infty$ and a Banach space X, $C^m([0,T];X)$ denotes the space of functions that are m times continuously differentiable in t with respect to the norm of X, and $L^2(0,T;X)$ is the space of functions with the norm $\int_0^T \|u(t)\|_X^2 dt$ being finite.

For any function space described above, we say that a vector valued function belongs to the function space if each of its components does.

Now we state our main theorem regarding the solvability of (1.1).

Theorem 2.1 *For $T > 0$ and natural number $m \geq 4$, if the initial filament $\boldsymbol{x}_0$ satisfies $\boldsymbol{x}_0 \in H^m(\mathbf{T})$ and $|\boldsymbol{x}_{0\xi}| \equiv 1$, and the external flow $\boldsymbol{F}$ satisfies $\boldsymbol{F} \in C([0,T];W^{m,\infty}(\mathbf{R}^3))$, then there exists $T_0 \in (0,T]$ such that a unique solution $\boldsymbol{x}(\xi,t)$ of (1.1) exists and satisfies*

$$\boldsymbol{x} \in C([0,T_0];H^m(\mathbf{T})) \cap C^1([0,T_0];H^{m-2}(\mathbf{T}))$$

The above theorem gives the time-local unique solvability of (1.1). We note that Nishiyama proved the existence of the solution in $C([0,T];H^2(\mathbf{T}))$ for any $T > 0$, and comparing with our result, we notice that the case $m = 3$ is missing. So far, we don't know whether solvability can be shown in this case.

3 Construction of Solution

In this section, we give a brief explanation regarding the construction of the solution. The method shown in this section is due to Nishiyama [10]. We construct the solution to problem (1.1) by passing to the limit $\varepsilon \to +0$ in the following regularized problem.

$$
(3.1) \qquad \begin{cases} \boldsymbol{x}_t = -\varepsilon \boldsymbol{x}_{\xi\xi\xi\xi} + \dfrac{\boldsymbol{x}_\xi \times \boldsymbol{x}_{\xi\xi}}{|\boldsymbol{x}_\xi|^3 + \varepsilon^\alpha} + \boldsymbol{F}(\boldsymbol{x}, t), & \xi \in \mathbf{T},\ t > 0, \\ \boldsymbol{x}(\xi, 0) = \boldsymbol{x}_0(\xi), & \xi \in \mathbf{T}, \end{cases}
$$

where $\varepsilon > 0$ and α with $0 < \alpha < 8/3$ are real parameters. The solution of problem (3.1) can be constructed by an iteration scheme based on the solvability of the following linear problem.

$$
(3.2) \qquad \begin{cases} \boldsymbol{x}_t = -\varepsilon \boldsymbol{x}_{\xi\xi\xi\xi} + \boldsymbol{G}, & \xi \in \mathbf{T},\ t > 0, \\ \boldsymbol{x}(\xi, 0) = \boldsymbol{x}_0(\xi), & \xi \in \mathbf{T}. \end{cases}
$$

Finally, the unique existence of the solution to (3.2) in $C\big([0,T]; H^m(\mathbf{T})\big) \cap C^1\big([0,T]; H^{m-2}(\mathbf{T})\big)$ for any $T > 0$ and $m \geq 2$ is known from the standard theory of parabolic equations. Hence, by iteration, we can prove the solvability of problem (3.1) in the same function space. It is shown in [10] that a solution of (3.1) belonging to $C\big([0,T]; H^2(\mathbf{T})\big)$ satisfies $|\boldsymbol{x}_\xi(\xi, t)| \geq c_0 > 0$ for some positive constant c_0 for all $\xi \in \mathbf{T}$ and $t \in [0,T]$. We also make use of this property in the next section.

4 Energy Estimates of the Solution

Our next and final step is to derive energy estimates for the solution to (3.1) which are uniform with respect to $\varepsilon > 0$. This will allow us to pass to the limit $\varepsilon \to +0$ and finish the proof of Theorem 2.1. We do this by deriving suitable energies that allow us to estimate the solution in the appropriate function space. The derivation of such energy is the most important part of the proof and thus, we go into more detail. For simplicity, we derive energy estimates for the solution to our original problem (1.1) because the arguments for the uniform estimates of the solution to (3.1) are the same.

Our objective is to derive energy estimates for the solution of

$$
(4.1) \qquad \begin{cases} \boldsymbol{x}_t = \dfrac{\boldsymbol{x}_\xi \times \boldsymbol{x}_{\xi\xi}}{|\boldsymbol{x}_\xi|^3} + \boldsymbol{F}(\boldsymbol{x}, t), & \xi \in \mathbf{T},\ t > 0, \\ \boldsymbol{x}(\xi, 0) = \boldsymbol{x}_0(\xi), & \xi \in \mathbf{T}, \end{cases}
$$

belonging to $C\big([0,T]; H^m(\mathbf{T})\big) \cap C^1\big([0,T]; H^{m-2}(\mathbf{T})\big)$ on some time interval $[0, T_0]$ with $T_0 \in (0, T]$. The difficulty arises from the fact that a solution of (4.1) stretches, i.e. $|\boldsymbol{x}_\xi| \not\equiv 1$ even if $|\boldsymbol{x}_{0\xi}| \equiv 1$. When $|\boldsymbol{x}_\xi| \equiv 1$, many useful properties of the solution can be utilized to obtain energy estimates, but these properties are not at our disposal in the present problem setting.

To overcome this, we modify the Sobolev norm to obtain a suitable form of energy which allow us to derive the necessary estimates. First, we set $\boldsymbol{v} := \boldsymbol{x}_\xi$ and take the ξ derivative of (4.1) to rewrite the equation in terms of $\boldsymbol{v}$.

$$
\begin{cases} \boldsymbol{v}_t = f\boldsymbol{v} \times \boldsymbol{v}_{\xi\xi} + f_\xi \boldsymbol{v} \times \boldsymbol{v}_\xi + (\mathrm{D}\boldsymbol{F})\boldsymbol{v}, & \xi \in \mathbf{T},\ t > 0, \\ \boldsymbol{v}(\xi, 0) = \boldsymbol{v}_0(\xi), & \xi \in \mathbf{T}, \end{cases} \tag{4.2}
$$

where we have set $\boldsymbol{v}_0 := \boldsymbol{x}_{0\xi}$, $f = 1/|\boldsymbol{v}|^3$, and omitted the arguments of $\boldsymbol{F}$. Since the energy estimate for the solution in $C\big([0,T]; H^2(\mathbf{T})\big)$ is already obtained in Nishiyama [10], we only show the higher order estimates. Following standard procedures, we differentiate the equation in (4.2) with respect to ξ, k times for a fixed k satisfying $3 \le k \le m$ and set $\boldsymbol{v}^k := \partial_\xi^k \boldsymbol{v}$ to obtain

$$
\boldsymbol{v}^k_t = f\boldsymbol{v} \times \boldsymbol{v}^k_{\xi\xi} + kf\boldsymbol{v}_\xi \times \boldsymbol{v}^k_\xi + (k+1) f_\xi \boldsymbol{v} \times \boldsymbol{v}^k_\xi + \boldsymbol{G}^k, \tag{4.3}
$$

where $\boldsymbol{G}^k$ are terms that contain derivatives of $\boldsymbol{v}$ up to order k. From here on, we regard terms with derivatives of $\boldsymbol{v}$ up to order k as lower order and disregard the precise expression of the terms. We can do this because these terms are harmless in terms of regularity when estimating the solution, although the nonlinearity of these terms are high in general and cause the estimates to become time-local. In what follows, we will use the symbol $\sim$ to denote that two sides are equal modulo lower order terms such as $\boldsymbol{G}^k$. For example, (4.2) can be expressed as

$$
\boldsymbol{v}^k_t \sim f\boldsymbol{v} \times \boldsymbol{v}^k_{\xi\xi} + kf\boldsymbol{v}_\xi \times \boldsymbol{v}^k_\xi + (k+1) f_\xi \boldsymbol{v} \times \boldsymbol{v}^k_\xi.
$$

Now that we have derived (4.3), the standard method would be to take the inner product of $\boldsymbol{v}^k$ and (4.3) and integrate over $\mathbf{T}$ with respect to ξ to estimate the time evolution of $\|\boldsymbol{v}^k\|$. This is not possible for our equation because the terms with derivatives of $\boldsymbol{v}^k$ cause a loss of regularity. To avoid such loss, we employ a series of change of variables to derive a modified energy from which we can derive the necessary estimates. The key idea is to decompose $\boldsymbol{v}^k$ into two parts. More precisely, we decompose $\boldsymbol{v}^k$ as

$$
\boldsymbol{v}^k = \frac{(\boldsymbol{v} \cdot \boldsymbol{v}^k)}{|\boldsymbol{v}|^2}\boldsymbol{v} - \frac{1}{|\boldsymbol{v}|^2}\boldsymbol{v} \times (\boldsymbol{v} \times \boldsymbol{v}^k). \tag{4.4}
$$

The above decomposes $\boldsymbol{v}^k$ into the sum of its $\boldsymbol{v}$ component and the component orthogonal to $\boldsymbol{v}$. The decomposition is well-defined since we know that $|\boldsymbol{v}| \ge c_0 > 0$. The principle part of the components are $\boldsymbol{v} \cdot \boldsymbol{v}^k$ and $\boldsymbol{v} \times \boldsymbol{v}^k$ respectively, and we define two new variables

$$
h^k := \boldsymbol{v} \cdot \boldsymbol{v}^k,
$$

$$
\boldsymbol{z}^k := \boldsymbol{v} \times \boldsymbol{v}^k,
$$

and estimate them separately.

4.1 Estimate of h^k

We first derive an equation for h^k. Taking the inner product of $\boldsymbol{v}$ and equation (4.2) yields

$$\boldsymbol{v}\cdot\boldsymbol{v}_t = \boldsymbol{v}\cdot\big((\mathrm{D}\boldsymbol{F})\boldsymbol{v}\big).$$

Differentiating k times with respect to ξ further yields

$$\boldsymbol{v}\cdot\boldsymbol{v}_t^k + k\boldsymbol{v}_\xi\cdot\boldsymbol{v}_t^{k-1}\sim 0.$$

Since we are estimating the solution in H^m with $m\geq 4$, we can regard $\|v\|_{W^{3,\infty}(\mathbf{T})}$ as lower order, and thus, we can further calculate

$$\begin{aligned}0 &\sim \boldsymbol{v}\cdot\boldsymbol{v}_t^k + k\boldsymbol{v}_\xi\cdot\boldsymbol{v}_t^{k-1}\\ &\sim [\boldsymbol{v}\cdot\boldsymbol{v}^k + k\boldsymbol{v}_\xi\cdot\boldsymbol{v}^{k-1}]_t\\ &= [h^k + k\boldsymbol{v}_\xi\cdot\boldsymbol{v}^{k-1}]_t.\end{aligned}$$

Finally, since $k\boldsymbol{v}_\xi\cdot\boldsymbol{v}^{k-1}$ is lower order, we obtain

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}\|h^k + k\boldsymbol{v}_\xi\cdot\boldsymbol{v}^{k-1}\|^2 \leq C(1+\|v\|_k)^{n(k)}\|v\|_k^2,$$

where $n(k)$ is an integer depending on k that is greater than 0 in general. From the above estimate, we see that there is a $T_1\in(0,T]$ such that for some constant $C_*>0$ depending on $\|v_0\|_k$ and T_1, h^k satisfies

$$\|h^k(t)\|^2\leq C_*$$

for any $t\in(0,T_1]$.

4.2 Estimate of $\boldsymbol{z}^k$

Next we consider $\boldsymbol{z}^k$. Directly calculating the t derivative of $\boldsymbol{z}^k=\boldsymbol{v}\times\boldsymbol{v}^k$ yields

$$\boldsymbol{z}_t^k \sim f\boldsymbol{v}\times\boldsymbol{z}_{\xi\xi}^k + (k-2)f\boldsymbol{v}\times(\boldsymbol{v}_\xi\times\boldsymbol{v}_\xi^k) + (k+1)f_\xi\boldsymbol{v}\times\boldsymbol{z}_\xi^k \tag{4.5}$$

First we notice that

$$\begin{aligned}\boldsymbol{v}\times(\boldsymbol{v}_\xi\times\boldsymbol{v}_\xi^k) &= (\boldsymbol{v}\cdot\boldsymbol{v}_\xi^k)\boldsymbol{v}_\xi - (\boldsymbol{v}\cdot\boldsymbol{v}_\xi)\boldsymbol{v}_\xi^k\\ &\sim h_\xi^k\boldsymbol{v}_\xi - (\boldsymbol{v}\cdot\boldsymbol{v}_\xi)\boldsymbol{v}_\xi^k.\end{aligned}\tag{4.6}$$

To proceed further, we must express $\boldsymbol{v}_\xi^k$ in terms of h^k and $\boldsymbol{z}^k$. Specifically, we apply the decomposition as in (4.4) and obtain

$$\begin{aligned}\boldsymbol{v}_\xi^k &= \frac{(\boldsymbol{v}\cdot\boldsymbol{v}_\xi^k)}{|\boldsymbol{v}|^2}\boldsymbol{v} - \frac{1}{|\boldsymbol{v}|^2}\boldsymbol{v}\times(\boldsymbol{v}\times\boldsymbol{v}_\xi^k)\\ &\sim \frac{h_\xi^k}{|\boldsymbol{v}|^2}\boldsymbol{v} - \frac{1}{|\boldsymbol{v}|^2}\boldsymbol{v}\times\boldsymbol{z}_\xi^k.\end{aligned}$$

Substituting this into (4.6) yields

$$\begin{aligned} \boldsymbol{v} \times (\boldsymbol{v}_\xi \times \boldsymbol{v}_\xi^k) &\sim h_\xi^k \boldsymbol{v}_\xi - (\boldsymbol{v} \cdot \boldsymbol{v}_\xi)\boldsymbol{v}_\xi^k \\ &\sim h_\xi^k \left(\boldsymbol{v}_\xi - \frac{(\boldsymbol{v} \cdot \boldsymbol{v}_\xi)}{|\boldsymbol{v}|^2}\boldsymbol{v} \right) + \frac{(\boldsymbol{v} \cdot \boldsymbol{v}_\xi)}{|\boldsymbol{v}|^2}\boldsymbol{v} \times \boldsymbol{z}_\xi^k \\ &= -\frac{h_\xi^k}{|\boldsymbol{v}|^2}[\boldsymbol{v} \times (\boldsymbol{v} \times \boldsymbol{v}_\xi)] + \frac{(\boldsymbol{v} \cdot \boldsymbol{v}_\xi)}{|\boldsymbol{v}|^2}\boldsymbol{v} \times \boldsymbol{z}_\xi^k \end{aligned}$$

Substituting this back into (4.5) yields

$$\boldsymbol{z}_t^k \sim f\boldsymbol{v} \times \{\boldsymbol{z}_{\xi\xi}^k - (k-2)\frac{h_\xi^k}{|\boldsymbol{v}|^2}\boldsymbol{v} \times \boldsymbol{v}_\xi\} + \left\{(k-2)f\frac{(\boldsymbol{v} \cdot \boldsymbol{v}_\xi)}{|\boldsymbol{v}|^2} + (k+1)f_\xi\right\}\boldsymbol{v} \times \boldsymbol{z}_\xi^k.$$

Next we focus on first term on the right-hand side

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